1000 Thomas Jefferson Street NW
Washington, DC 20007-3835
202-403

Overview

The Common Core State Standards (CCSS) in mathematics establish rigorous expectations for all learners, including English language learners (ELLs). Although these standards present challenges, they create opportunities to more fully incorporate ELLs into standards-based reform.

The CCSS in mathematics include a focus on the mathematical content required for students at each grade level and also include Standards for Mathematical Practice that apply in different ways across all grade levels. The eight Standards for Mathematical Practice are the following:

1. Make sense of problems and persevere in solving low1 203.9 38.2

Name of Prototype Lesson	Grade	Module	Lesson
Make Series of Longer Than And Shorter Than Comparisons	Kindergarten	3	3
Use Visual Models to Add and Subtract Two Fracti with the Same Units	4	5	16

written language skills; building background knowledge; clarifying content delivered in a second
wledge.

Teach Academic Vocabulary

In the lessons that follow, vocabulary is selected for instruction because it is important for

purposeful explanation of a scenario, illustration, or other information that is not part of the

(There are 24 hours in one day.)

Teacher Modeling and Explanation. Teacher explanation and modeling of thought processes, of the manner in which lesson activities should be carried out, and of high-quality responses will be particularly beneficial for ELLs because explanation and examples enhance comprehension. Explanation and modeling should be used to support students before they are struggling, with teachers clearly explaining each task and modeling an expected student response. Modeling all mathematical discourse that students will be using in their own collaboration with peers and in writing will be beneficial for ELLs at all levels.

Cueing. At the beginning of the lesson, include a clear focus on stating the standards, objectives, and agenda for the day, communicated in student-friendly language. This provides an advance schema for the students and allows them to begin to anticipate how the new information will connect to previous learning. In addition, cueing provides reinforcement of key vocabulary. Cueing is recommended for use at the beginning of lessons and again at the end (Wiliam, 2011).

&DSLWDOL]H RQ 6WXGHQW¶V +RPH /DQJXDJH 6NLOOV DQG .

The scaffolds in the previous sections may be helpful to all students. Scaffolds unique to ELLs include those that capitalize on their home language knowledge and skills to help them acquire knowledge and skills in a new language. A large body of research indicates that ELLs draw on conceptual knowledge and skills acquired in their home language in learning their new language (Dressler, 2006) and that instructional methods that help ELLs draw on home language knowledge and skills promote literacy development in a new language (August et al., 2009; Carlo et al., 2004; Liang, Peterson, & Graves, 2005). I

levels ofproficiency. ELLs at all levels of proficiency have access to scaffolds that provide multiple means of representation, action and expression, and engagement. Because teachers generally have more than one level of ELL in a group and because within a level of proficiency there are individual differences in knowledge and skills, teachers should reduce the scaffolding to meet the unique needs of individual students.

Conventions Used to Describe AIR Scaffolding

The purpose of the prototypes is to provide illustrative examples of new activities and additional supports to the original lessons that are beneficial to ELLs. Each prototype includes scaffolds described in the previous section, but with specific connections to the content and intended goals of the lessons. This may include references to specific resources, explanation of specific activities, and scripting of specific teacher language. *AIR New Activity* refers to an activity not in the original lesson that AIR has inserted into the original lesson. For example, Comparing Cueing Make Series of Longer Than and Shorter Than Comparisons is a new activity. *AIR Additional Supports* refer to additional supports added to a component already in place in the original lesson. *AIR new activities* and *AIR additional supports* are boxed whereas the text that is in the original lesson is generally not boxed. *AIR Routines for Teachers* are activities that include instructional conversations that take place between teachers and students. In the *AIR Routines for Teachers* the text of the original lessons appears in standard black, whereas the AIR additions or supports to the lessons are in green.

Kinsella, K. (2012). *Evidence-based principles to guide English language development in the Common Core Standards era*. Retrieved from http://teacher.scholastic.com/products/teaching-english-language-learners/pdf/Kinsella_ELD_CCSS_4-19-12Handout.pdf

Lesaux, N. K., Kieffer, M. J., Faller, S. E., & Kelley, J. G. (2010). The effectiveness and ease of implementation of an academic vocabulary intervention for linguistically diverse students in urban middle schools. *Reading Research Quarterly, 45*(2), 196 228. doi:10.1598/RRQ.45.2.3

Lesaux, N. K., Kieffer, M. J., Kelley, J. G., & Harris, J. (in press). Effects of academic vocabulary instruction for linguistically diverse adolescents: Evidence from a randomized field trial. *American Educational Research Journal*.

Liang, L. A., Peterson, C. A., & Graves, M. F. (2005). Investigating two approaches to fostering *Reading Psychology, 26*(4 5), 387 400.

Moschkovich, J. (2014). *Supporting ELLs in mathematics: Mathematics tasks with annotations and other resources for implementing the Common Core State Standards. Developed for the Understanding Language Initiative*. Stanford, CA: Stanford University Graduate School of Education. Retrieved from http://ell.stanford.edu/teaching_resources/math

Ryoo, K. (2009). *Learning science, talking science: The impact of a technology-enhanced* F X U U L F X O X P R Q V W X G H Q W V ¶ *lyDiverse mainstream classrooms* Q J L Q O L (Doctoral dissertation). Retrieved from http://gradworks.umi.com/33/64/3364450.html

Silverman, R., & Hines, S. (2009). The effects of multimedia-enhanced instruction on the vocabulary of English-language learners and non-English-language learners in pre-kindergarten through second grade. *Journal of Educational Psychology, 101*(2), 305 314. doi:10.1037/a0014217

Thompson, A., & Radosavljevic, A. (2013). Fostering mathematical discourse with Socratic seminars. In M. Civi BT s

objective and agenda in student-friendly language) and to provide an initial introduction to the key vocabulary of the lesson. This provides an advance schema for the students and allows them to begin to anticipate how the new information will connect to previous learning. The purpose of this cueing is to establish what the lesson will be about and to help students know where to focus their attention throughout the lesson. Because this section is not included in the original lesson, subsequent times allotted for activities have been modified.

AIR Routine for Teachers

Write on board and read aloud: *Today I will compare the length of different objects. I will use the words* ³O R Q J H U W K D Q ´ D Q G ³V K R U W H U W K D Q ´ W R H [S O D L Q Z K D

T: A few days ago, we compared the height

length is related to how long something is. Length, long. Say it with me length, long. I can length long. Length, long.

When we compared two things, we noticed what was the same and what was different. Today, we compare. The word compare. (Compare.) Now say it to your neighbor: compare. (Compare.) Today you will compare different objects and use the words

to show they swam away. How many fish did not (teacher shakes head left to right, indicatir
swim away?

S: 5.

T: Now this time circle (teacher draws circle in the air) a group of 2. Circle another 2.

S: (Circle two groups of 2.)

T: How many (teacher pantomimes question) fish have you circled so far?

T: Circle 1 more. Now how many are circled?

T:

Notes on Multiple Means of Representation:

Your below grade level students will benefit from extra practice in determining what objects are longer than and shorter than in order to be ready for comparing two different lengths with a third object in this lesson.You can use interactive technology such as that found at http://www.kidport.com/Gradek/math/MeasureGeo/MathK_Tall.htm

AIR Additional Supports
Background knowledge for students and key academic terms At times, students may need access to background knowledge before they can comfortably begi on a lesson. In this example, the background knowledge for students and key academic terms ar included to ensure that students have access to the foundational information required for work in lesson.
Concrete and visual models For students at the entering, emerging, and transitioning levels of English proficiency, concrete and visual models can make mathematical concepts more apparent and accessible. These models may include manipulatives, illustrations, or other opportunities to have hands-on experiences with the concepts. Some of the nonessential vocabulary words in this lesson are easily taught during the application problem, the words *monkey* and *banana* are used, and to best teach these words, the use of photographs, vid in the case of the banana, realia, are best. Although the previous lesson used comparatives and superlatives (like tall, taller, tallest), ELLs at the entering, emerging, and transitioning levels may need a review of these terms and how they are related. Review a visual like the one immediately following to reinforce these ideas.

AIR Routine for Teachers

is not in Standard English and with ELLs working to learn English.
Scaffolded language At times, the linguistic complexity of the language impedes student access to the content being To clarify the key concepts and maintain rigor while providing access to the content, it may be necessary to reword some text using present tense, shorter sentences, fewer clauses, and contexts familiar to students. To provide access to the content of these oral instructions (making a series of comparisons of longer than and shorter than items), using a modified version (see the example that follows) will benefit ELLs at the entering, emerging, and transitioning levels. Note that these instructions will replace the instructions on the original sheet.
AIR Routine for Teachers

(Say to students) I am going on a trip. I will miss my family.

Draw a picture as I tell you about my home so that I can take it with me.

- Draw a house in the middle of the paper. Make it the size of your finger. (Gesture to show the middle of the paper and which finger you want the house to be equal to.)
- Draw my daughter. She is shorter than the house. (Gesture to the word wall card or objects you have been using throughout the lesson while also emphasizing the academic vocabulary is shorter than.)

Activity Sheet 3 Activ

Grade 4, Module 5, Lesson 16: Use Visual Models to Add and Subtract Two Fractions With

284(sru84(tha)4(t 2IR h(r)s insnfo)ert</Ar)3n[ELLs

AIR Lesson Introduction

AIR New Activities
Background knowledge for teachers Background knowledge for teachers is provided as a way to help teachers become more familiar with the educational contexts their students may have experienced before beginning in U.S. schools. T important to help teachers tailor instruction and assessments to ensure that all students are appropriate challenged and supported. School systems outside the United States may not emphasize operations with fractions as we do in t United States, and instruction may not involve fraction computation until secondary school. Students unfamiliar with fractions should have opportunities to make connections between the area, the set model, and the distance model for fractional parts.
Cueing The lesson opens with cueing to provide an anticipatory overview for students (in the form of an objecti and agenda in student-friendly language) and to provide an initial introduction to the key vocabulary of the lesson. This provides an advance schema for the students and allows them to begin to anticipate ho new information will connect to previous learning. The purpose of this cueing is to establish what the will be about and to help students know where to focus their attention throughout the lesson. Further, this lesson begins with clear introductions to key academic vocabulary so that students will be able to more quickly access the content of the lesson. Because this section is not included in the original lesson, subsequent times allotted for activities have been modified
Key academic vocabulary Introducing new vocabulary should be explicit and should take into account words that are homo (like *sum* and *some*), words that have multiple functions (like the word *number* which can be used as a noun, a verb, and adjective), and words that are often difficult for students to accurately hear or pronounce (like *eighths*). Students should be provided with structures to record their new vocabulary, such as graphic organizers, a specific note-taking format, or a student-created illustrated dictionary. ELLs should be given opportunities to review vocabulary to which they have been exposed but n have committed to memory. For example, the modified version of this lesson opens with a brief rev of the word *compare* with partners using flash cards or a folded graphic organizer they have created. They could review vocabulary by lesson or in other ways, such as by grammatical form (nouns, verbs, phrases). The cards or folded graphic organizer could have the vocabulary word and perhaps an illustration on the fr the back could contain a definition, a first-language translation, an exemplary sentence, and questi that would engage the students in discussion about the words. ELLs should have structured opportunities to use this key academic vocabulary, both new terms and previously learned terms across all four modalities (speaking, reading, writing, and listening) each
AIR Routine for Teachers
Introduce objectives, student outcomes, and key vocabulary for the lesson. Display the standard associated with this lesson. Write on board and read aloud. T: I will use pictures and manipulatives to show how to add and subtract fractions with the units. T: In our last lesson, you compared fractions to see what was similar and what was different.

Try not to look at the board.

S: – – – – – – –

T: (Point to –.) 4 eighths is the same as 1 of what unit?

S: –

T: (Beneath –, write –.) Count by 1 eighths again. This time, convert to and 1 whole. Try not to look at the board.

For ELLs, use the term *simplify* instead of the word *convert* (which is related to units of measure and is taught elsewhere in the curriculum).

S: – – – – – – – –

T: What other fractions can we simplify?

S: – and –.

T: (Point to –.)

S: –.

T: (Beneath –, write

AIR Additional Supports

Teacher modeling and explanation

ELLs may not initially understand instructions given by the teacher and may benefit from modeli
help reinforce the directions.

To further reinforce these ideas for ELLs, after practicing chorally, engage the students in a
count around the room. Counting with the students and beginning with fourths, the first student (

AIR Additional Supports

Key academic vocabulary and recording and processing key ideas

Although fluent speakers of English may already be familiar with key academic vocabulary, ELLs may benefit from instruction to explain and add depth to the meanings. Further, ELLs may need clear parameters for recording and processing their ideas, which may not be familiar practices. Recording main concepts and ideas can deepen understanding and increase retention, positioning ELLs to be

Note: This fluency activity reviews G4–M5 Lesson 15.

T: On your boards, draw two area models. (Allow students time to draw.)

T: (Write ___.) Partition (which is the same as wide) your first diagram into an area model that shows ___. Then, write ___ beneath it.

S: (Partition first area model into 2 equal units. Shade one unit. Write ___ beneath it.)

T: (Write ___ ___ ___.) Partition your second area model to show ___. Then, write ___ beneath it.

S: (Partition second area model into 5 equal units. Shade 2 units. Write ___ beneath the shaded area.)

T: Partition the area models so that both fractions have common denominators.

S: (Draw dotted lines through the area models.)

T: Write a greater than, less than, or equal sign to compare the fractions.

AIR Routine for Teachers
Clarifying language Keisha ran mile in the morning and mile in the afternoon. Which distance was longer? Did Keisha run farther in the morning or in the afternoon? Solve independently. Share your solution with your partner. Did your partner solve the problem in the same way or a different way? Explain.
Sentence starters To scaffold the speaking and writing of ELLs, sentence starters or sentence frames can be very supportive and provide structures for students to use in communicating their thinking. Using sentence starters is a simple way to help ELLs structure their thinking and create meaningful sentences with increasing sophistication. First, I ___. Then, I _____. Next, I _____. Finally, I _____. Model it with another example that is so students have an example of how to use it.
Visual support Students may need a visual to support their understanding of . It would be beneficial to show a variety of solutions. Use a number line or tape diagram to show distance.

Note: This application problem builds on the concept development of G4–M5, Lessons 14 and 15, where students learned to compare fractions with unrelated denominators by finding common units.

Common Core Inc. Concept Development inls0 g [(Note:)] TJ ET BT 0 54150 1721 72 396.58

Note:

sixths.

S: 1 sixth!

T: Draw one arrow above the number line to model – . (Demonstrate.) Tell me the subtraction sentence.

S: – – – .

Repeat with – – .

T: Solve for 7 sixths 2 sixths. Work with a partner. Use the language of units and subtraction.

During these tasks, ELLs should be paired with English proficient peers to facilitate engagement in academic conversations in English. It also would be beneficial to reinforce the concept of equivalence by always showing multiple representations of the problems.

This lesson assumes that students know that the right side of the number line represents larger numbers than those on the left. This might not be known or clear to all students, particularly those who may have literacy in a language that reads right to left.

Background knowledge for students

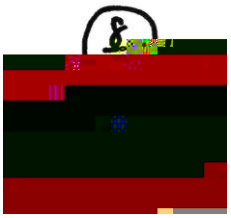
Because ELLs may have attended schools outside the United States, or may have not fully learned content from previous grades, building background knowledge for students can be an essential form of scaffolding. In the previous lesson, students were asked to compare two fractions, noting similarities and differences. Because the word *difference* was recently used with a meaning unlike that intended in this context.

Problem 2: Decompose to record a difference greater than 1 as a mixed number

AIR Additional Support s

[redacted]

Although students may have heard the verb *decompose* in a previous lesson, review the definition for clarity.



Problem 3: Solve for the sum using unit language and a number line.

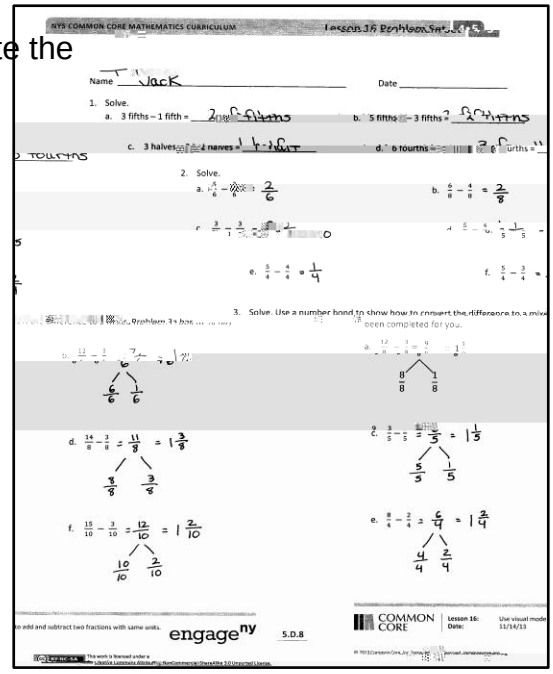
T: Look back at the first example. (Point to the number line representing 5 sixths + 4 sixths.)

T: Count as we add. 1 sixth, 2 sixths, 3 sixths, 4 sixths. Where are we now?

S:

Common Core Inc. Problem Set

Students should do their personal best to complete the Problem Set within the allotted 10 minutes.


NYS COMMON CORE MATHEMATICS CURRICULUM

Name Jack Date

1. Solve.
a. 3 fifths – 1 fifth = 2 fifths
b. 5 fifths – 3 fifths = 2 fifths
c. 3 halves – 2 halves = 1 half
d. 6 fourths = 2 wholes

2. Solve.
a. $\frac{4}{6} - \frac{2}{6} = \frac{2}{6}$
b. $\frac{6}{8} - \frac{4}{8} = \frac{2}{8}$
e. $\frac{5}{4} - \frac{4}{4} = \frac{1}{4}$
f. $\frac{5}{4} - \frac{3}{4} =$

3. Solve. Use a number bond to show how to convert the difference to a mixed number. Problem 3 has been completed for you.

$\frac{12}{8} - \frac{3}{8} = \frac{9}{8} = 1\frac{1}{8}$ (8/8, 1/8)

$\frac{11}{6} - \frac{5}{6} = \frac{6}{6} = 1$ (6/6, 1/6)

d. $\frac{14}{8} - \frac{3}{8} = \frac{11}{8} = 1\frac{3}{8}$ (8/8, 3/8)

c. $\frac{9}{5} - \frac{4}{5} = \frac{5}{5} = 1\frac{1}{5}$ (5/5, 1/5)

f. $\frac{15}{10} - \frac{3}{10} = \frac{12}{10} = 1\frac{2}{10}$ (10/10, 2/10)

e. $\frac{8}{4} - \frac{2}{4} = \frac{6}{4} = 1\frac{2}{4}$ (4/4, 2/4)

engage^ny 5.D.8

COMMON CORE Lesson 16: Date: 11/14/13

Name_____ Date _____

1. Solve.

   a. 3 fifths 1 fifth = _____

   b. 5 fifths 3 fifths = _____

   c. 3 halves 2 halves = _____

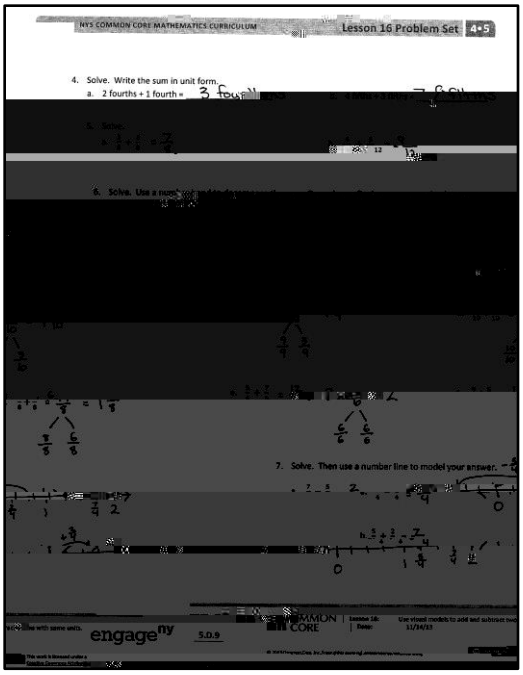
   d.

AIR Additional Supports

Key academic vocabulary

For ELLs, the term *number bond* may not be familiar from previous lessons, and students may need instruction or a reminder of what it means. Focus student attention on the verb *convert,* which in this case, means to rewrite a fraction greater than 1 as a mixed number.

Consider identifying one or two key problems from each section of the worksheet for ELLs and allowing them to show their thinking in different ways (with number lines, tape diagrams, blocks, etc.)



AIR Additional Supports
Structured opportunities to speak with a partner or small group ELLs at the entering, emerging or transitioning levels may need additional language support to fully participate in the class discussion.
Sentence frames Post the following sentence frames and rehearse their use by reading through them as a class and practicing different terms that could fill the blanks. ƒ The _____ model helps me add fractions because _____. ƒ Number bonds help to decompose fractions into mixed numbers because _____. ƒ When adding fractions, it is important to remember to _____.

Common Core Inc. Exit Ticket

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work today and plan more effectively for future lessons. You may read the questions aloud to the students.

AIR Additional Supports
Key academic vocabulary and cueing Clarifying key academic vocabulary and cueing ELLs so that they are familiar with lessons objectives will support ELLs.
AIR Routine for Teachers

T: combined tional Support

Homework

AIR Additional Supports
Homework scaffold Providing scaffolded homework assignments can provide the teacher with information about the level of depth to which students understand content. Further, it can be useful to echo the format of the lesson in the homework, as is included in the additional that follows, instructing students to use a number line to illustrate their thinking. You may scaffold the assignment to allow students to focus on a smaller number of problems in greater depth and provide the teacher with formative data on what students understand. Teachers may also assign one or two of each type of problem.

Grade 8, Module 3, Lesson 6: Proofs of

Common Core, Inc. Lesson Introduction

Student Outcomes

Students extend the previous laws of exponents to include all integer exponents.

Students base symbolic proofs on concrete examples to show that $(xy)^n = x^n y^n$ is valid for all integer exponents.

Lesson Notes

This lesson is not designed for all students, but for those who would benefit from a lesson that enriches their existing understanding of the Laws of Exponents. For that reason, this is an optional lesson that can be used with students who have demonstrated mastery over concepts in Topic A.

AIR Additional Supports
Background knowledge for teachers It can be useful for teachers to consider certain aspects of the content as related to previous exp ELLs may have had. This background information for teachers can help teachers consider which scaffolds for instruction and assessment may be most appropriate for each student. To successfully participate in the Socratic discussion described in this lesson, students at the entering, emerging, transitioning, and expanding levels of English proficiency must have access to a scaffolde version the text (noted in what follows) and information at an appropriate reading level in advance prepare for the discussion. Therefore, the lesson described here will require a homework assignment in the previous class meeting to allow students time to prepare for the Socratic discussion.
AIR New Activity
Cueing, introduction of objectives, student outcomes, and key vocabulary for

exponent and what it means.

T: Remember the term integer that we have been using in this unit. Remember that integers are whole numbers that are positive, negative, or zero. Fractions and decimals are not integ Some examples of integers are four, negative four, and zero. Tell your partner three mo integers.

Introduce t

Statements	Reasons
Given _____	Given _____
If m = 0, the left side of the equation is _____	Because _____ *or* (By definition of _____)
If m= 0, the right side of the equation is _____	Because _____ (

(iv) $a, b < 0$

- Ask students why there are no other possibilities.
- Ask students if we need to prove case (i).
- *No, because* (A) *corresponds to case (i) of (11).*

We will prove the three remaining cases in succession.

AIR Additional Supports
Background knowledge for teachers, key academic vocabulary, and clarification of key concepts Because students will not have seen this source material in advance, students at the entering, emerging, and transitioning levels may be unprepared to participate in a genuine Socratic discussion of this content. This activity is best reworked by having students focus on the meanings of the different cases in pairs and engage in a whole-group discussion and explanation.
AIR Routine for Teachers
With an assigned partner, students identify a and b on the coordinate plane for each case (i)–(iv).

The left side and the right side are equal, thus, *case (ii)* is done.

Case (iii): We have to prove that for any positive , ( ) , when the integers and satisfy and . This is very similar to case *(ii)*, so it will be left as an exercise.

Students complete Exercise 4 independently or in pairs.

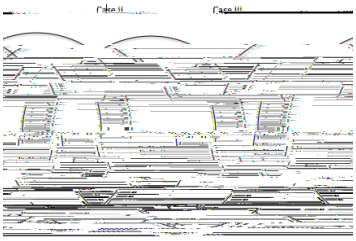
Exercise 4

Proof of Case (iii): Show that when and , ( ) is still valid. Let for some positive integer . Show that the left side and right sides of ( ) are equal.

The left side is

( ) ( )

( ) *By* — *for any whole number*



Clarification of key concepts
Following the protocol established in examining cases ii and iii, the creation of a partial T proofs is useful here. To further enrich the experience of the students at all levels of English proficiency, should be invited to make conjectures about what will happen. T: Look at case vi (a, b < 0). Find it on your coordinate plane. Which quadrant is it in?
AIR Routine for Teachers
Structured opportunities to speak with a partner or small group T: Based on what we just did with the last cases, what do you think we should do with this Turn and talk with your neighbor. Students will mention substitution of integers for variables and using a chart to organize the proofs. T: Work with your partner to complete as much of this proof as you can. After students have had some time to work, match pairs of students together. T: Now you and your partner are going to get together with another pair, to make a group of fo students. Share what you have been working on, and listen carefully as your new group members explain their thinking. Then review as a whole class to ensure that students have successfully completed the pr
Background knowledge for teachers and seen in the lesson plan that follows and in the completed proof.

Common Core Inc. Closing

Summarize, or have students summarize, the lesson.
Students have proven that the Laws of Exponents are valid for any integer exponent

Common Core Inc. Exit Ticket

9. Show directly that for any positive integer , .
10. Show directly that for any positive integer , ( )

AIR Additional Supports
Clarification of key concepts and use of a graphic organizer or foldable to organize ideas Have students complete a graphic organizer (below) with a partner to help them synthesize ideas and write independently. With a partner, complete the following 2-3-1 summary graphic organizer. Use the word

———— *By* *for whole numbers* *and* *(6)*
(—)

———
(—)

(—) *By* — *for any whole number* *(B)*

4. Prove for any positive number,

— — *By definition*

——— *By the Product Formula for Complex Fractions*

——— *By*

AIR Additional Supports
Clarification of key concepts For the problem set, ELLs may benefit from clarification of key concepts to make the meaning of the problems more comprehensible and streamlined. This may include ensuring that the context is familiar for all students and school-oriented, using present tense, and breaking lengthy sentences into shorter sentences. For example, substitute this text for problem 1: Tell seven friends a funny joke. Each friend tells your joke to five of their friends. Then each of those five friends tells the joke to five more people. No one heard the joke more than one time. How many people (not including you) will hear the joke? Express your answer in exponential notation.

Algebra I, Module 3, Lesson 5: The Power of Exponential Growth

Overview

The following table outlines the scaffolds that have been added to support ELLs throughout the Common Core Inc. Lesson 5, The Power of Exponential Growth.

AIR New Activity refers to an activity not in the original lesson that AIR has inserted into the original lesson. *AIR Additional Supports* refer to additional supports added to a component already in place in the original lesson. *AIR new activities* and *AIR additional supports* are boxed whereas the text that is in the original lesson is generally not boxed. *AIR Routines for Teachers* are activities that include instructional conversations that take place between teachers and students. In the *AIR Routines for Teachers* the text of the original Common Core Inc. lessons appears in standard black, whereas the AIR additions to the lessons are in green.

Original Component by Common Core, Inc.	AIR Additional Supports	AIR New Activities Type of Scaffold
None included		Cueing
None included		Key academic vocabulary Graphic organizers or foldables
Opening exercise	Clarification of key concepts	
Example 1	Clarification of key concepts Background knowledge for students Key academic vocabulary	
Discussion	Writing Structured opportunities for students to speak with a partner or small group	
Example 2	*No changes recommended*	
Exercise 1	Use of multimedia to enhance comprehension Background knowledge for teachers	
Exercise 2	Clarification of key concepts	

Patterns and Sequence Sort

Have the students complete a sort activity. This formative assessment will provide the teacher with background information from a previous lesson on arithmetic and geometric sequences and also provide students an opportunity to review these concepts.

Directions

- Enlarge, copy on cardstock, laminate, and cut out all the cards (below) for each pair or group.
- Give one set to each group and have them sort the cards into three groups (Arithmetic Seque Geometric Sequence, or Neither).
- The teacher will walk around the room and monitor the groups. fo0635] TJ152] TJ1g
- fo175q34.2 ET

Common Core Inc. Opening Exercise

Direct students to begin the lesson with the following comparison of two options.

Two equipment rental companies have different penalty policies for returning equipment late:

Company 1: On day , the penalty is 5. On day 2, the penalty is . On day , the penalty is 5. On day , the penalty is 2 and so on, increasing by 5 each day the equipment is late.

Company 2: On day , the penalty is . On day 2, the penalty is . On day , the penalty is . On day , the penalty is 8 and so on, doubling in amount each addi

Rental Company Late-Fee Policy

Company 1

AIR Additional Supports

Clarification of key concepts, background knowledge for students, and key academic vocabulary

The language load in this example is quite heavy, using nonessential words in less common ways, the term *ruler* being used to describe a leader, *store* being used to describe a warehouse, and low-frequency words like *modest*. If possible, students should preview the picture book, *One Grain of Rice: A Mathematical Folktale*

Common Core Inc. Discussion Writing Exponential Formulas

Ask students to consider how the exponential expressions of Example 1, part (b) relate to one another.

- Why is the base of the expression 2?
  Since each successive square has twice the amount of rice as the former square, the factor by which the rice increases is a factor of 2.
- What is the explicit formula for the sequence that models the number of rice grains in each square? Use to represent the number of the square and to represent the number of rice grains assigned to that square.
  ( ) 2( ), where ( ) represents the number of rice grains belonging to each square, and represents the number of the square on

Common Core Inc. Exercise 2

Exercise 2

A rare coin appreciates at a rate of 2 a year. If the initial value of the coin is 5 , after how many years will its value cross the mark? Show the formula that will model the value of the coin after years.

The value of the coin will cross the mark between and years;
( ) ( ) .

AIR Additional Supports
Clarification of key concepts and background knowledge for students To familiarize ELLs with the concepts and content of this problem, reworking the wording o problem (without sacrificing rigor or content) is essential.
AIR Routine for Teachers

T: The verb appreciate mean has several meanings. In mathematics class, it means that somethi increases in value over time. The amount it appreciates (or incr

a =
b =

	( ) =	Think about what happens to each of these sequences when a = 0 and b > 1.		( ) = +
0			0	
1			1	
2			2	

AIR Additional Support s

Structured opportunities to speak with a partner or small group

Have students work with their shoulder partner to compare their findings. As a whole group, dis the big idea (exponential functions grow faster than arithmetic functions)

Common Core Inc. Exit Ticket Sample Solutions

Chain emails are emails with a message suggesting you will have good luck if you forward the email on to others. Suppose a student started a chain email by sending the message to friends and asking those friends to each send the same email to more friends exactly day after they received it.

a. Write an explicit formula for the sequence that models the number of people who will receive the email on the day. (Let the first day be the day the original email was sent.) Assume everyone who receives the email follows the directions.

( ) ,

b. Which day will be the first day that the number of people receiving the email exceeds ?

On the th day.

Common Core Inc. Problem Set Sample Solutions

1. A bucket is put under a leaking ceiling. The amount of water in the bucket doubles every minute. After 8 minutes, the bucket is full. After how many minutes is the bucket half full?

   minutes

2. A three-bedroom house in Burbville was purchased for

Alternate Problem Set

The italicized text that follows can be used in place of the text for the problems as indicated in the original lesson. Problem 1 would be made more accessible through the use of an image of a bucket, because this word (bucket) is not typically among those first learned by ELLs.

Problem 1:

Include an image of a bucket.

Problem 2

Natalia buys a house for $190,000. Each year, the value of the house will increase by 1.8%. Write a formula to model the price of the house in t years. Find the price of the house in 5 years.

Problem 3

In the year 1999, 924 students graduated from a university. Every year, the university increases the number of graduates by a factor of . . What formula models this situation? Approximately how many students will graduate in 2014?

Problem 4

In the year 2001, the population of New York City was 8,008,288 *This number is . % greater than the population of New York City in the year 2000. In what year would the population of New York City be more than 10 million people if the growth rate stays at 2.1%? Write a formula to show your answer.*

Problem 5

In the year 2013, 959 million smartphones were sold. This number of was an increase of . % from the number sold in the year 2012. How many smartphones will be sold in 2018 if the growth rate is 32.7% every year? Do you think smartphones sales will continue to grow at this rate? Why or why not? Explain your thinking.

Problem 6

Jack and Meg are having a concert. They want lots of people to attend. The concert is in 7 days. Jack and Meg have different ideas (strategies) to tell people about the concert:

Jack passes out fliers a day for days.

On the first day, Meg tells of he U I U L H Q G V D E R X W W K H F R Q F H U W 2 Q *friends tells 10 more people about the concert. On the third day, each person tells another friend about the concert. This pattern continues for days.*

a. *How many people know about the concert u* V L Q J - D F N ¶ V V W U D W H J \ N Q R Z D E R X W W K H F R Q F H U W X V L Q J 0 H J ¶ V V W U D *strategy.*

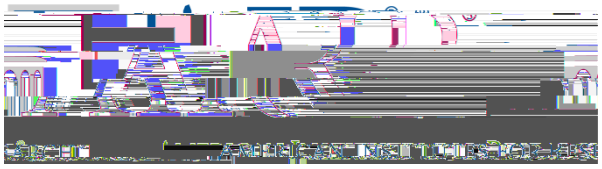
b. 2 Q Z K L F K G D \ G R H V 0 H J ¶ V V W U D W H J \ U H D F K P R

c. *How can Meg change her strategy to reach more people than Jack does over 7 days?*

LOCATIONS

Domestic

Washington, D.C.



1000 Thomas Jefferson Street NW
Washington, DC 20007-3835
202.403.5000 | TTY 877.334.3499

www.air.org